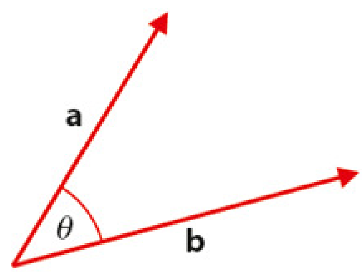
9-4 The Scalar Product of Two Vectors

The Scalar Product of Two Vectors

The Scalar product of vectors $\vec{a}$ and $\vec{b}$ denoted by $\vec{a} \cdot \vec{b}$ is defined as the product of the magnitudes of the vectors times the cosine of the angle between them

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$$



* Note – the result is a scalar NOT a vector. This is commonly called a “dot product.”

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Properties of the Scalar Product

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$$

$$\vec{a}\cdot\left(\vec{b}+\vec{c}\right)=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$$

$$\vec{a}\cdot\vec{a}=\left|\vec{a}\right|^2$$

$|\vec{a}|\cdot|\vec{a}|\cos 0$

$|\vec{a}|^2$

$$k\left(\vec{a}\cdot\vec{b}\right)=k\vec{a}\cdot\vec{b}=\vec{a}\cdot k\vec{b}$$

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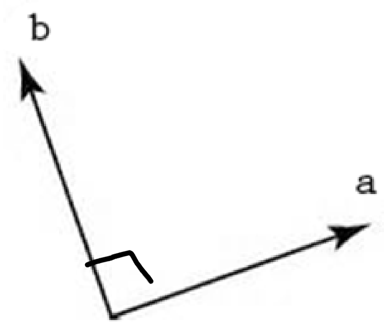

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos 90°$$

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot 0$$

$$\vec{a}\cdot\vec{b}=0$$

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If two vectors are expressed in component form,

$$\vec{u} = u_1\mathbf{i} + u_2\mathbf{j}$$
$$\vec{v} = v_1\mathbf{i} + v_2\mathbf{j}$$

then $$\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2$$

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Law of Cosines

$$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}|\cdot|\vec{v}|\cos\theta$$

$$|\vec{u}-\vec{v}|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$$

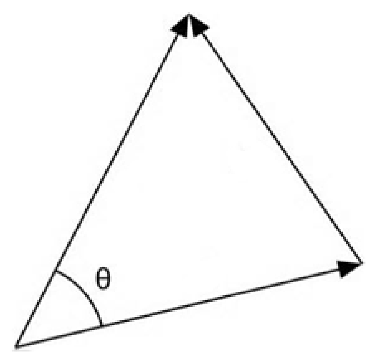
$$|\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}|\cdot|\vec{v}|\cos\theta = \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$$

$$|\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}|\cdot|\vec{v}|\cos\theta = |\vec{u}|^2 - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + |\vec{v}|^2$$

$$-2|\vec{u}|\cdot|\vec{v}|\cos\theta = -\vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u}$$

$$-2|\vec{u}|\cdot|\vec{v}|\cos\theta = -2\vec{u}\cdot\vec{v}$$

$$|\vec{u}|\cdot|\vec{v}|\cos\theta = \vec{u}\cdot\vec{v}$$



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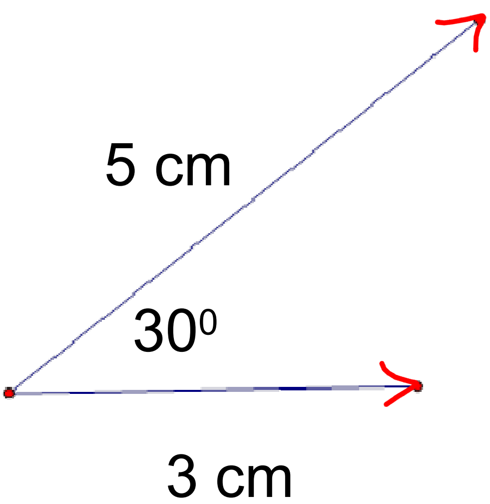
Ex1. Find the dot product $\vec{u} \cdot \vec{v}$

a.)



$\cos(30) = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} \cdot 5 \cdot 3 = \frac{15\sqrt{3}}{2}$

b.) $\vec{u} = 3\mathbf{i} + 4\mathbf{j}$
$\vec{v} = -2\mathbf{i} + 6\mathbf{j}$

$3 \cdot (-2) + 4 \cdot 6$

$-6 + 24 = 18$

c.) $\vec{u} = \langle 3, 1 \rangle$

$\vec{v} = \left\langle -\frac{2}{3}, 2 \right\rangle$

$\cancel{3} \cdot \frac{-2}{\cancel{3}} + 1 \cdot 2$

$-2 + 2 = 0$

Angle Between Two Vectors

Since $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$, then

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{a_1 \cdot b_1 + a_2 \cdot b_2}{\sqrt{(a_1)^2 + (a_2)^2} \cdot \sqrt{(b_1)^2 + (b_2)^2}}$$

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Ex2. Find the angle between these vectors $\vec{u} = 2\mathbf{i} + 5\mathbf{j}$

$\vec{v} = -6\mathbf{i} + -3\mathbf{j}$

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$

$\theta = \cos^{-1}\left(\dfrac{-12 - 15}{\sqrt{29} \cdot \sqrt{45}}\right)$

$\theta = 138.368$

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